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Vertex Polynomial for the Splitting Graph Of Comb and Crown

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Abstract – The vertex polynomial of the graph $G = (V, E)$ is defined as $V(G, x) = \sum_{k=0}^{\Delta(G)} v_k x^k$, where $\Delta(G) = \max\{d(v)/v \in V\}$ and v_k is the number of vertices of degree k. In this paper we derived the vertex polynomial for splitting graph of Comb, Crown, $P_n \odot \bar{K}_m, (n \geqslant 2), C_n \odot \bar{K}_m, (n \geqslant 3)$, their union and their sum.

Index Terms – Comb, Crown, Splitting graph, Vertex Polynomial, Union, Sum.

1. INTRODUCTION

In a graph $G = (V, E)$, we mean a finite undirected, non-trivial graph without loops and multiple edges. The vertex set is denoted by V and the edge set by E. For $v \in V$, $d(v)$ is the number of edges incident with v, the maximum degree of G is defined as $\Delta(G) = \max\{d(v)/v \in V\}$. For terms not defined here, we refer to Frank Harary[3]. For each vertex v of a graph G, take a new vertex v', join v' to all the vertices of G adjacent to v. The graph S(G) thus obtained is called splitting graph of G [2]. The graph $G = (V, E)$ is simply denoted by G. Let G_1 and G_2 be two graphs, the union $G_1 \cup G_2$ is defined to be (V, E) where $V = V_1 \cup V_2$ and $E = E_1 \cup E_2$, the sum $G_1 + G_2$ is defined as $G_1 \cup G_2$ together with all the lines joining points of V_1 to V_2. The graph obtained by joining a single pendent edge to each vertex of a path is called Comb. Any cycle with pendant edge attached to each vertex is called Crown.

2. VERTEX POLYNOMIAL FOR THE SPLITTING GRAPH OF COMB

Definition: 2.1

The graph obtained by joining a single pendent edge to each vertex of a path is called Comb.

Theorem: 2.2

Let G be a Comb with order $2n, (n \geqslant 2)$. The vertex polynomial of S(G) is

$V(S(G), x) = (n-2)x^6 + 2x^4 + (n-2)x^3 + (n+2)x^2 + nx, n \geqslant 2.$

Proof:

Let G be a comb with $2n\ (n \geqslant 2)$ vertices. Therefore, S(G) have $4n\ (n \geqslant 2)$ vertices. Among $2n$ vertices of G, n vertices are pendent vertices; among remaining n vertices, $n-2$ have degree 3 and 2 vertices have degree 2. In S(G), each new vertex corresponding to each vertex of V has same degree as in V of G and rest of vertices becomes twice the degree, gives the result.

Example: 2.3

Take $n = 3$ in the above theorem. We have the graph

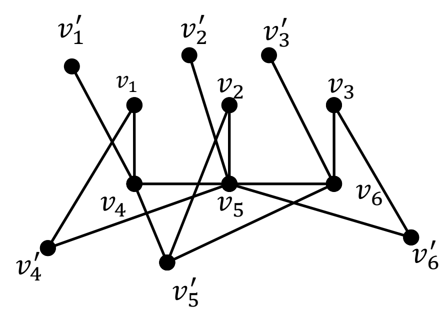

Figure 1

Here, $V(S(G), x) = x^6 + 2x^4 + x^3 + 5x^2 + 3x$.

Theorem: 2.4

Let G be a Comb with order $2n, (n \geqslant 2)$ and $\zeta = S(G) \cup S(G) \cup \ldots \cup S(G)$ (m times). Then the vertex polynomial is $V(\zeta, x) = m(n-2)x^6 + 2mx^4 + m(n-2)x^3 + m(n+2)x^2 + mnx, n \geqslant 2, m \geqslant 1$.

Proof:

Consider m copies of S(G), here the number of vertices of S(G) increased by m copies but degree of each vertex remains unchanged. Therefore, each coefficient of the vertex polynomial of S(G) multiplied by m gives the result.

Theorem: 2.5

Let G be a Comb with order $2n, (n \geqslant 2)$. The vertex polynomial of $mS(G)$ is $V(mS(\zeta), x) = m(n-2)x^{6+4n(m-1)} + 2mx^{4+4n(m-1)} + m(n-2)x^{3+4n(m-1)} + m(n+2)x^{2+4n(m-1)} + mnx^{1+4n(m-1)}, n \geqslant 2, m \geqslant 1$.

Proof:

In $mS(G)$, each vertex degree of ζ has increased by $4n(m-1)$ gives the required result.

3. VERTEX POLYNOMIAL FOR THE SPLITTING GRAPH OF CROWN ($C_n \odot K_1$)

Definition: 3.1

Any cycle with pendant edge attached to each vertex is called Crown.

Theorem: 3.2

Let G be a Crown with order $2n, (n \geqslant 3)$. Then the vertex polynomial of $S(G)$ is given by $V(S(G), x) = nx^6 + nx^3 + nx^2 + nx, n \geqslant 3$.

Proof:

Let G be a Crown with order $2n$, $(n \geqslant 3)$. Therefore, $S(G)$ have $4n$ $(n \geqslant 3)$ vertices. Among $2n$ vertices of G, n vertices are pendant vertices; remaining n vertices have degree 3. In $S(G)$, each new vertex corresponding to each vertex of V has same degree as in V of G and rest of vertices becomes twice the degree, gives the result.

Example: 3.3

Take $n = 3$ in the above theorem. We have the graph

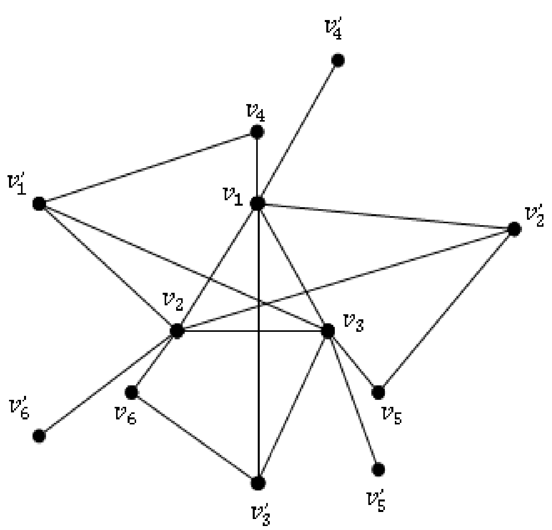

Figure 2

Here, $V(S(G), x) = 3x^6 + 3x^3 + 3x^2 + 3x$.

Theorem: 3.4

Let G be a Crown with order $2n, (n \geqslant 3)$. The vertex polynomial of $\zeta = S(G) \cup S(G) \cup \dots \cup S(G)(m \text{ times})$ is $V(\zeta, x) = nmx^6 + nmx^3 + nmx^2 + nmx$,

$n \geqslant 3, m \geqslant 1$.

Theorem: 3.5

Let G be a Crown with order $2n, (n \geqslant 3)$. The vertex polynomial of $mS(G)$ is given by $V(mS(G), x) = nmx^{6+4n(m-1)} + nmx^{3+4n(m-1)} + nmx^{2+4n(m-1)} + nmx^{1+4n(m-1)}, n \geqslant 3, m \geqslant 1$.

4. VERTEX POLYNOMIAL FOR THE SPLITTING GRAPH OF $P_n \odot \overline{K}_m, (n \geqslant 2)$

Theorem: 4.1

Let G be $P_n \odot \overline{K}_m$. Then the vertex polynomial of $S(G)$ is given by $V(S(G), x) = (n-2)x^{2(m+2)} + 2x^{2(m+1)} + nmx^2 + (n-2)x^{m+2} + 2x^{m+1} + nmx, n \geqslant 2, m \geqslant 1$.

Proof:

Let G be $P_n \odot \overline{K}_m$. We can observe that, $n-2$ vertices have degree $m+2$, 2 vertices have degree $m+1$ and nm vertices have degree 1.

In $S(G)$, each new vertex corresponding to each vertex of V has same degree as in V of G and rest of vertices becomes twice the degree shows the required result.

Example: 4.2

Consider the Graph $P_2 \odot \overline{K}_2$. Then the corresponding graph is illustrated as follows;

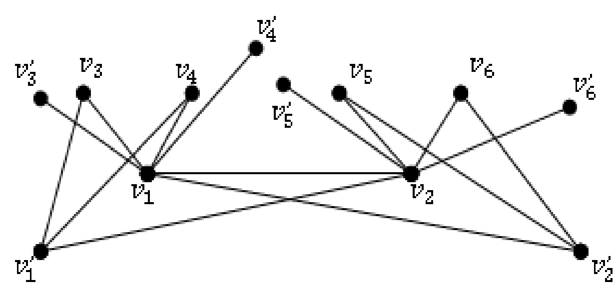

Figure 3

Here, $V(S(G), x) = 2x^6 + 2x^3 + 4x^2 + 4x$.

Theorem: 4.3

Let G be $P_n \odot \overline{K}_m$. Then the vertex polynomial of $\zeta = S(G) \cup S(G) \cup \dots \cup S(G)(k \text{ times})$ is $V(\zeta, x) = (n-2)kx^{2(m+2)} + 2kx^{2(m+1)} + nmkx^2 + (n-2)kx^{m+2} + 2kx^{m+1} + nmkx, n \geqslant 2, m \geqslant 1$.

Theorem: 4.4

Let G be $P_n \odot \overline{K}_m$. Then the vertex polynomial of kS(G) is given by $V(S(G), x)$

$$= (n-2)x^{2(m+2)+2n(k-1)(m+1)} + 2x^{2(m+1)+2n(k-1)(m+1)} + nmx^{2+2n(k-1)(m+1)} + (n-2)x^{m+2+2n(k-1)(m+1)} + 2x^{m+1+2n(k-1)(m+1)} + nmx^{1+2n(k-1)(m+1)}, n \geqslant 2, \quad m \geqslant 1.$$

5. VERTEX POLYNOMIAL FOR THE SPLITTING GRAPH OF $C_n \odot \overline{K}_m, (n \geqslant 3)$

Theorem: 5.1

Let G be $C_n \odot \overline{K}_m, (n \geqslant 3)$. Then the vertex polynomial of S(G) is given by $V(S(G), x) = nx^{2(m+2)} + nmx^2 + nx^{m+2} + nmx, \ n \geqslant 3$.

Proof:

Let G be $C_n \odot \overline{K}_m$. We can observe that, n vertices have degree $m+2$ and nm vertices have degree 1. In S(G), each new vertex corresponding to each vertex of V has same degree as in V of G and rest of vertices becomes twice the degree shows the required result.

Example: 5.2

Consider the Graph $C_3 \odot \overline{K}_1$. Then the corresponding graph is depicted as follows;

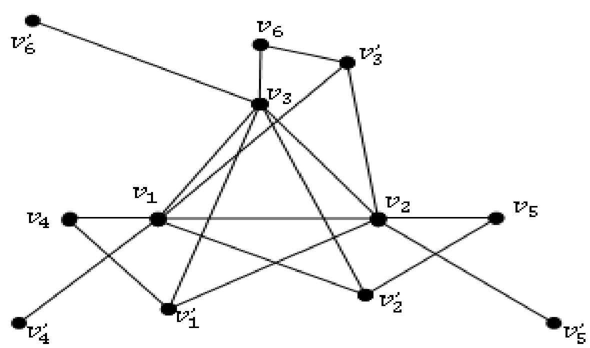

Figure 4

Here, $V(S(G), x) = 3x^6 + 3x^3 + 3x^2 + 3x$.

Theorem: 5.3

Let G be $C_n \odot \overline{K}_m, (n \geqslant 3)$. The vertex polynomial of $\zeta = S(G) \cup S(G) \cup \ldots \cup S(G)$ (k times) is $V(\zeta, x) = nkx^{2(m+2)} + nmkx^2 + nkx^{m+2} + nmkx, \ n \geqslant 3$.

Theorem: 5.4

Let G be $C_n \odot \overline{K}_m, (n \geqslant 3)$. Then the vertex polynomial of kS(G) is given by $V(S(G), x) = nkx^{2(m+2)+2n(k-1)(1+m)} + nmkx^{2+2n(k-1)(1+m)} + nkx^{m+2+2n(k-1)(1+m)} + nmkx^{1+2n(k-1)(1+m)}, \ n \geqslant 3$.

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