Various Techniques for Frequency Offset Estimation in OFDM System: A Survey

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Abstract – The rapid growth in the wireless technology possess the need of high data rate transmission. This can be accomplished by using Orthogonal Frequency Division Multiplexing (OFDM). It is a particular case of multicarrier modulation (MCM), where high rate data stream is transmitted over low rate subcarriers. The system uses a number of carriers to transmit data simultaneously. The disadvantage of this system is its sensitivity to frequency offset and multipath delay spread as the spectra of OFDM signal is not strictly band limited which causes ICI (inter carrier interference) and ISI (inter symbol interference), resulting in loss of orthogonality between the subcarriers and thereby degrade the system performance. This paper presents various important techniques to estimate the frequency offset in OFDM system.

Index Terms – Frequency offset; Inter carrier interference (ICI); Inter symbol interference (ISI); Cyclic Prefix (CP); cross-correlation; Kalman Filter; Orthogonal Frequency Division Multiplexing (OFDM).

1. INTRODUCTION

Wireless communications is the most important development of an era that has an extremely wide range of applications from TV remote to cellular phones. Wireless signals transmitted over the air usually experience frequency selective fading, i.e. different frequency components are faded by the channel differently. Complex equalization techniques are used to reduce the frequency-selective fading in single carrier (SC) systems. The frequency response of the equalizer is the inverse of frequency response of the channel, therefore, infinite numbers of equalizer taps are required. Signal noises can also be increased through the equalizer when a deep fade occurs. Therefore in conventional SC systems, even with the finest equalizer, a deep fade can still occur, which causes communication link failure.

In 1967 [1], first proposal to use parallel data transmission to minimize frequency selective fading channels was published, in which small numbers of sub-channels use carriers that fall within each deep-faded frequency band, error correcting codes can be used to recover the data along those corrupted sub-channels. Moreover in mid 1960s, the idea of using frequency division multiplexing and parallel data transmission was published [2]. In conventional parallel transmission, as shown in Fig.1.(a), the whole frequency band is shared by ten non-overlapping sub-channels which is not a bandwidth efficient transmission. In the 1960s, with the use of overlapping sub-channels spectral efficiency was improved, as shown in Fig.1. (b), which saves half of the spectrum as compared to conventional parallel transmission. At this stage, OFDM system came into existence, which is frequency multiplexing method that maintains orthogonality among sub-carriers.

Fig.1.(a)Traditional non-overlapping MCM;(b) orthogonal MCM. [3]

It was considered for high density recording, high speed modems, as well as digital mobile communications, in 1980s. Finally in 1990s, it was utilized for wide band data communication over mobile radio Frequency modulation channels, wireless local area network IEEE802.11 a / g standard, high bit rate digital subscriber lines, very high speed digital subscriber lines, digital audio broadcasting (DAB), asymmetric digital subscriber lines and high definition television (HDTV) terrestrial broadcasting [4].

In this paper, Section I describes the introduction to OFDM, Section II gives the details of OFDM, frequency offset, Section III describes system model, Section IV explains the techniques to estimate the frequency offset and Section V summarizes the discussion.

2. OFDM SYSTEM

In this system, the data stream is divided amid large number of closely spaced sub-carriers, which justify the frequency division multiplex portion of the OFDM name. Data is transferred in a parallel form, instead of transmitting in serial
form. OFDM system can be expounded as a type of multicarrier modulation in which sub-carrier spacing is chosen carefully so that each subcarrier be orthogonal to another. Basically orthogonality is attained by carefully selecting carrier spacing, such that carrier spacing must be equal to the reciprocal of the useful symbol period. It can also be attained by precise frequency and time synchronization before demodulation at the receiver side. If it is not achieved, ISI and ICI occur which degrade the signal to interference noise ratio (SINR). Unlike frequency division multiplexing, the carriers in an OFDM signal are organized so that the sidebands of the individual carriers overlap which results in negligible adjacent carrier interference in received signals, it also ensures bandwidth efficiency. IFFT and FFT functions are needed for the modulation at the transmitter and demodulation at the receiver of OFDM systems. The employment of DFT to replace the banks of sinusoidal generator and the demodulation significantly reduces the implementation complexity of OFDM.

Advantages of OFDM system-
- Proficiently implementing using Fast Fourier Transform (FFT) which replaces the stock of sinusoidal generator and reduces the accomplishment complication.
- The spectral efficiency is high due to overlapping of carriers, as compared to other double sideband modulation schemes, spread spectrum.
- Simple equalization, each sub-channel is almost flat fading, so it only needs a one-tap equalizer to overcome channel effect.
- Immune to frequency selective fading.

Disadvantages of OFDM system-
- Sensitive to frequency offset.
- High Peak to Average Power Ratio (PAPR).
- The problem of frequency and symbol synchronization occurs.

2.1. Frequency offset

Frequency offset, i.e. the carrier frequency of the receiver is different from the carrier frequency of the transmitter or frequency mismatch between the transmitter and receiver, which comes from many sources such as Doppler shift or frequency drifts in the modulator and the demodulator oscillators. When relative motion between transmitter and receiver exist, Doppler shift takes place and is given by,

\[ \Delta f = \frac{v}{c} f_c \]  

Where, \(v\) is relative velocity, \(c\) is speed of light and \(f_c\) is the carrier frequency.

When this happens, the received baseband signal, instead of being centered at 0 MHz will be centered at frequency \(f_0\). Fig.2. shows the frequency offset problem. OFDM systems are more delicate to frequency errors than SCM systems. One of the harmful effects of frequency offset is loss of orthogonality, which causes the inter carrier interference (ICI) among sub carriers. This offset also causes reduction of sinc amplitude because the sinc is no longer sampled at the peak frequency. This implies frequency offset synchronization is very important to OFDM system.

Fig.2. Frequency offset. [5]

3. OFDM SYSTEM MODEL

Fig.2. shows the block diagram of the OFDM system, in this system a wide-band channel is divided into \(N\) orthogonal narrow-band sub-channels. OFDM modulation and demodulation are implemented using \(N\) point IFFT an FFT respectively. At transmitter, message bits are mapped into a series of BPSK or QAM symbols which are then converted into \(N\) parallel bit stream. Each of \(N\) symbols is modulated on the different sub-carriers after serial-to-parallel conversion.

Fig.3. OFDM system

By means of Quadrature Amplitude Modulation (QAM) or Phase Shift Keying (PSK) serial data stream is mapped into constellation symbols and the resultant symbol is blocked into
N data vector. Using IFFT, the \( t^{th} \) data vector \( x_t = [x_t(0), x_t(1) ... x_t(N-1)] \) is multiplexed into N sub carriers to provide the time domain samples,

\[
v_t(n) = \frac{1}{N} \sum_{k=0}^{N-1} x_t(k) \exp\left(-j\frac{2\pi}{N}kn\right), 0 \leq n \leq N - 1
\]

Where, \( v_t = [v_t(0) v_t(1) ... v_t(N-1)] \) is the \( t^{th} \) time domain OFDM block for \( i = 0, 1, ..., \infty \). At the receiver for a single path channel, the discrete complex-baseband OFDM signal \( r(n) \) is given as,

\[
r(n) = v(n)\exp\left(j\frac{2\pi nN}{N}\right) + g(n)
\]

Where, \( g(n) \) = noise

\( \varepsilon = \) frequency offset

4. FREQUENCY OFFSET ESTIMATION TECHNIQUES

In this section some frequency offset estimation techniques have been disputed. Frequency offset estimation strains the inceptive analog approximation of the carrier frequency. Estimation techniques can be pilot based and non-pilot based, pilots for example pseudo random series or null symbols are employed to decide the commence of an OFDM symbol.

4.1. Cyclic Prefix (CP) based frequency offset estimation

In this technique, with perfect symbol synchronization, frequency offset consequences phase rotation of \( \frac{2\pi nN}{N} \) in the received signal. With negligible channel effect, the phase difference between CP and the corresponding rear part of an OFDM symbol (spaced N samples apart) is \( \frac{2\pi nN}{N} = 2\pi \varepsilon \) [6]. Then frequency offset can be found from the phase angle of the product of CP and the corresponding rear part of an OFDM symbol.

Frequency offset using CP based,

\[
\dot{\varepsilon} = \left(\frac{1}{2\pi}\right) \arg\{r^*[n]r[+N]\}
\]

\( n = -1, -2, ..., -N_c, \) where \( N_c = \) length of CP. In order to reduce the noise effect, its average can be considered over the samples in a CP interval.

\[
\dot{\varepsilon} = \left(\frac{1}{2\pi}\right) \arg\left\{ \sum_{n=-N_c}^{-1} r^*[n]r[+N] \right\}
\]

arg( ) Performed \( tan^{-1}( ) \), the range of the frequency offset estimation is \([-0.5 + 0.5]\) and mean square error performed by \( \dot{\varepsilon} - \varepsilon \).

4.2. Symbol Based frequency offset estimation

Two identical training symbols are transmitted consecutively and the corresponding signals with frequency offset of \( \varepsilon \) are related with each other [6]. For an OFDM transmission symbol at one receiver with an assumption of the absence of noise the 2N point sequence is [7]

\[
y_n = \frac{1}{N} \sum_{k=0}^{N-1} H_k c_k e^{2j\pi(n+k)\varepsilon} \]

Where \( n = 0, 1, ... 2N - 1 \)

The \( k^{th} \) element of the N point DFT of the first N points (5) is,

\[
Y_{1k} = \sum_{n=0}^{N-1} y_n e^{-2\pi jkn/N}
\]

Where \( k = 0, 1, ... N - 1, \)

The second half of this sequence is

\[
Y_{2k} = \sum_{n=0}^{N-1} y_n N e^{-2\pi jkn/N}
\]

Where, \( y_n + N = y_n e^{2\pi j/N}, Y_{2k} = Y_{1k}e^{2\pi j/N} \), including the AWGN noise

\[
R_{1k} = Y_{1k} + G_{1k}
\]

\[
R_{2k} = Y_{1k}e^{2\pi j/N} + G_{2k},
\]

Where \( k = 0, 1, ... N - 1, \)

Observe that between the first and second DFT symbols, both ICI and signal are changed in exactly the same way, by a phase shift proportional to frequency offset. Therefore, if frequency offset \( \varepsilon \) is estimated using above observation, it is possible to obtain accurate estimation even when the offset is too large for satisfactory data demodulation [7].

\[
\dot{\varepsilon} = \left(\frac{1}{2\pi}\right) \tan^{-1}\left(\frac{\sum_{k=0}^{N-1} 1_m[R_{2k}R_{1k}^*]}{\sum_{k=0}^{N-1} 1_m[R_{2k}R_{1k}]}\right)
\]

The limit for accurate estimation by (8) is \(| \varepsilon | \leq 0.5\)

4.3. Training Sequence Based frequency offset estimation

In this scheme, estimation is found using training symbol that is repetitive with some shorter period. Frequency offset only within the range \(| \varepsilon | \leq 0.5 \), since frequency offset can be large
at initial synchronization stage, we need estimation techniques that can cover wider frequency offset range. The range of frequency offset can be increased by using training symbols that are repetitive with some shorter period. Let $D$ represents the ratio of the OFDM symbol length to the length of a repetitive pattern. Let the transmitter sends the training symbols with $D$ repetitive patterns in the time domain, which generated combo-type signal in the frequency domain after IFFT.

\[
\epsilon_b(k) = \begin{cases} 
B_m, & \text{if } k = D \cdot i, i = 0,1, \ldots, \left\lfloor \frac{N}{D} - 1 \right\rfloor \\
0, & \text{elsewhere}
\end{cases} \quad (10)
\]

Where $B_m$ represents an M-ary symbol and $N/D$ is an integer. After receiving repetitive length data sequence, receiver can make frequency offset estimation as [8]

\[
\hat{\epsilon} = \left(\frac{D}{2\pi}\right) \arg \left\{ \sum_{n=0}^{N} r^*[n]r[n] \right\} + \left(\frac{N}{D}\right) \quad (11)
\]

In this technique, the estimation range is $|\epsilon| \leq D/2$, which becomes wider as $D$ increases and number of samples for the computation of correlation is reduced by $1/D$, which degrade the mean square error (MSE) performance of the OFDM system.

4.4. Pilot based frequency offset estimation

Pilot tones are inserted in the frequency domain and transmit every OFDM symbol for frequency offset tracking. Using FFT the signals are transformed into $R_t[k]_{k=0}^{N-1}$ and $R_t+D[k]_{k=0}^{N-1}$, from which pilots are extracted. The signal is compensated with the estimated frequency offset in the time domain, after estimating the frequency offset from pilot tones in the frequency domain. In this process two different modes for estimation are implemented: acquisition and tracking modes. In the former, a large range of frequency offset including an integer frequency offset is estimated. The estimated integer frequency offset is estimated; on the other hand in later, only fine frequency offset is estimated. The estimated integer frequency offset is given by [9].

\[
\hat{\epsilon} = \left(\frac{1}{2\pi f_{sub}}\right) \max \left\{ \sum_{j=0}^{l-1} R_{t+D}[p[j], \epsilon]R^*[p[j], \epsilon] \times c_{t+D}[p[j]]c[p[j]] \right\} \quad (12)
\]

Where $L$ denotes the number of pilot tones, $p[j]$ denotes the location of the $j^{th}$ pilot tone, and $c[p[j]]$ denotes the pilot tone located at $p[j]$ in the frequency domain at the $j^{th}$ symbol period [5]. Fig.5. shows the Mean Square Error (MSE) vs. Signal to Noise Ratio (SNR) performance for CP based, Symbol based and Pilot based techniques, at $\epsilon = 0.15$. Both OFDM based and non-OFDM based pilot symbols can be used for frequency offset estimation and synchronization [15].

4.5. Blind frequency offset estimation

Blind estimation technique did not require any training symbol of pilot sub-carriers and performed well in frequency selective channels. This technique has low complexity due to use of minimum number of operations of multiplication and division. Maximum likelihood scheme is able to decode with probability close to one. It has fast convergence and achieves high accurate estimation [10].

The blind detection blind channel estimation based on the CP is that this estimation concept is standard-compliant and can be applied to all commonly used OFDM systems that use a CP. The blind detection without the necessity of pilot symbols for coherent detection is possible when joint equalization and detection is applied. This is possible by trellis encoding of differentially encoded modulated signals where decoding can be achieved by applying the Viterbi decoding.

![Fig.4. Blind system Model](image)

Typical algorithm structure includes stochastic gradient algorithm, recursive estimator, prediction error filtering, sub space algorithm and iterative techniques for maximum likelihood estimation. The main goal of a blind estimation is fast convergence to an operating point where the detection of information symbols is reliable as well as low computational complexity.

4.6. Moose method for frequency offset estimation

The Moose method is used for obtaining the frequency offset in the frequency domain [7]. This method is based on transmission of two consecutive equal OFDM symbols (here $z_{\ell,k}$ is the signal after the demodulation of the received signal). The offset value is calculated as,

\[
z_2[k] = z_1[k]e^{j2\pi \epsilon} \quad (13)
\]

Which is in the frequency domain corresponds to

\[
Z_2[i] = Z_1[i]e^{j2\pi \epsilon} \quad (14)
\]

Where $Z_p[i] = FFT(y_p[k]), p = 1,2$. Further, by (13) and (14) frequency offset can be calculated as,
\[ f_{FO} = \frac{1}{2\pi} \tan^{-1} \left( \frac{\sum_{i=0}^{N-1} R_i e^{j \pi N_i Z_i[i] Z_i[i]}}{\sum_{i=0}^{N-1} R_i} \right) \]  

(15)

The technique (15) is used for frequency offset range \(|f_{FO}| \leq \frac{\pi}{2\pi} = 0.5\), but it could be extended to larger values. This method assumes no data symbol transmission, so that this is suitable only for preamble period.

4.7. Clasen method

This method is proposed by Classen [11] which is based on pilots inserted in every symbol. In comparison to the Moose method this method allows the data symbol transmission. It is performed in two steps:

- The Coarse CFO estimation (large range of the CFO)
- The Fine CFO estimation (small range of the CFO)

Coarse synchronization is performed when the frequency offset is larger than one half of the subcarrier spacing. Main aim is to reduce the frequency offset within one half of the subcarrier spacing. Initially, two OFDM symbols \( r_i[k] \) and \( r_{i+D}[k] \) are transmitted. At the receiver side, FFT is performed and pilots are extracted (as in pilot based frequency offset estimation). The coarse CFO is estimated by,

\[ \hat{\epsilon} = \frac{1}{2\pi T_{sub}} \max \left\{ \sum_{j=0}^{N_p-1} R_{i+D}[p[j],e] \times R^*_i[p[j],e] c_{i+D}^*[p[j]] c_i[p[j]] \right\} \]

(16)

Where, D denotes the number of repetitive patterns of transmitted symbol, \( N_p \) denotes the number of pilots, \( p[j] \) denotes the location of the \( j \)th pilot tone, and \( c_i[p[j]] \) denotes the pilot tone located on the \( p[j] \)th subcarrier at the \( i \)th symbol. Before this step, fine frequency offset is estimated,

\[ \hat{\epsilon} = \frac{1}{2\pi T_{sub}} D \max \left\{ \sum_{j=0}^{N_p-1} R_{i+D}[p[j],\hat{\epsilon}] \times R^*_i[p[j],\hat{\epsilon}] c_{i+D}^*[p[j]] c_i[p[j]] \right\} \]

(17)

The final frequency offset represent sum of the coarse frequency offset estimation i.e. \( \hat{\epsilon} \) and fine frequency offset estimation i.e. \( \hat{\epsilon} \). After frequency offset estimation in two steps the compensation is performed in time domain. Fine synchronization follows the first step and estimates fractional part of the frequency offset.

4.8. Maximum Likelihood Estimation (MLE)

The signal energy and intercarrier interference energy both contribute coherently to the estimate, the algorithm generates extremely accurate estimates even when the offset value is great to demodulate the data values [12]. The estimation error is insensitive to channel spreading and frequency selective fading.

Adding the CP and frequency offset to the (1) equation, the \( q(n) \) vector is formed as,

\[ q^{cp}(n) = e^{j2\pi n}, v^{cp}(n) * h(n) + g(n) \]

(18)

After removing CP

\[ q^{cp}(n) = e^{j2\pi n}, v^{cp}(n) * h(n) + g(n) \]

\[ n = N_{cp}, \ldots, N_e - 1 \]

(19)

To estimate the frequency offset of \( q(n) \) vector, \( q_1 \) and \( q_2 \) vectors are formed,

\[ q_1(n) = e^{j2\pi N_{cp} N_e} (v(n) * h(n)); n = 0, \ldots, \frac{N}{2} - 1 \]  

(20)

And,

\[ q_2(n) = e^{j2\pi N_{cp} N_e} (v(n) * h(n)); n = 0, \ldots, \frac{N}{2} - 1 \]  

(21)

Assume \( p(n) = v(n) * h(n) \) and performing some calculations,

\[ q_2^{*} q_1 = e^{j2\pi (\frac{N}{2})} \left| p(n) \right|^2 \]  

(22)

\[ \Gamma = \frac{q_2^{*} q_1}{\left| q_2^{*} q_1 \right|} = e^{j2\pi (-\frac{N}{2})} = e^{j\pi N_e} = \cos \pi N_e - j \sin \pi N_e \]  

(23)

Finally the frequency offset estimation is,

\[ -\tan(\pi N_e T) = \frac{I_m(\Gamma)}{R_{e}(\Gamma)} \]

\[ \Rightarrow \hat{\epsilon} = \frac{1}{\pi} \tan^{-1} \left( -\frac{I_m(\Gamma)}{R_{e}(\Gamma)} \right) \]

(24)

4.9. Frequency offset estimation using cross-correlation

The method of CP auto-correlation cannot precisely estimate Doppler frequency in cell boundary. On the other hand, the method of frequency offset estimation using cross-correlation can precisely estimate Doppler frequency in cell boundary. This is because cross-correlation can regard other cell signal as
noise. User equipment should know the transmitted data from base station for cross-correlation. OFDM data symbols cannot be used for frequency offset estimation. Only known signals such as reference signal or synchronization signal can be used for estimation. For e.g., in 3G-LTE system, Synchronization channel (SCH) data can be used for known signal [13]. The cross-correlation value of received signal r(n) and synchronization signal u(n) is used for frequency offset estimation.

4.10. Extended Kalman Filter (EKF) method to estimate
Frequency offset

A state space model of the discrete Kalman filter is,

\[ z(n) = a(n)d(n) + v(n) \]

(25)

The observation z(n) has a linear relationship with the desired value d(n). Using discrete Kalman filter, d(n) can be recursively estimated based on the observation of z(n) and the updated estimation in each recursion is optimum in the minimum mean square sense [14].

\[ y(n) = x(n)e^{j2\pi n(n)/N} + w(n) \]

(26)

Observation y(n) is in a nonlinear relationship with the desired value ε(n), i.e.

\[ y(n) = f(\varepsilon(n)) + w(n) \]

(27)

Where, \( f(\varepsilon(n)) = x(n)e^{j2\pi n(n)/N} \)

(28)

In order to estimate \( \varepsilon(n) \) efficiently in computation, there is an approximate linear relationship using the first order Taylor’s expansion,

\[ y(n) \approx f(\dot{\varepsilon}(n-1)) \tilde{f}(\dot{\varepsilon}(n-1))[\varepsilon(n)\dot{\varepsilon}(n-1)] + w(n) \]

(29)

\( \dot{\varepsilon}(n-1) \) is the estimation of \( \varepsilon(n-1) \).

\[ \tilde{f}(\dot{\varepsilon}(n-1)) = \frac{\partial f(\varepsilon(n))}{\partial \varepsilon(n)} \bigg|_{\varepsilon(n) = \dot{\varepsilon}(n-1)} \]

\[ = j \left( \frac{2\dot{\varepsilon}(n-1)}{N} \right) x(n)e^{j \left( \frac{2\dot{\varepsilon}(n-1)}{N} \right)} \]

(30)

Define

\[ z(n) = y(n) - f(\dot{\varepsilon}(n-1)) \]

\[ d(n) = \varepsilon(n) - \dot{\varepsilon}(n) - 1 \]

(31)

And the following relationship

\[ z(n) = \tilde{f}(\varepsilon(n-1))d(n) + w(n) \]

(32)

Which is in the same form as equation (25), \( z(n) \) is linearly related to \( d(n) \). Hence the frequency offset \( \varepsilon(n) \) can be estimated in a recursive procedure similar to the discrete Kalman filter. As linear approximation is involved in the derivation, the filter is called the extended Kalman filter (EKF). The EKF provides a trajectory of estimation for \( \varepsilon(n) \). During iterations, error in each update decreases and the estimate becomes closer to the ideal value. Frequency offset can be mitigate by multiplying received symbol with complex conjugate of estimated offset and applying FFT.

Fig.5. MSE vs. SNR for three techniques i.e., CP-based, symbol by symbol and Pilot tone based. [6]

5. CONCLUSION

In this paper various frequency offset estimation techniques in OFDM system have been discussed. Pilot based frequency offset estimation is better than CP based and symbol based estimation, because it has superior MSE performance. The blind ML estimation of frequency offset in OFDM system did not require any training symbol of pilot subcarriers, performed well in frequency selective channel and has low complexity. The Moose method assumes no data symbol transmission, so it is suitable only for preamble period. On the other hand, Classen method allows the data symbol transmission. In maximum likelihood estimation of frequency offset, it has been shown that for small error in the estimate is conditionally unbiased. Furthermore, both the signal values and the ICI, contribute coherently to the estimate so it is possible to obtain very accurate estimates even when the offset is too high, this algorithm works well in the multipath channels. The method of CP auto-correlation cannot precisely estimate Doppler frequency. On the contrary, the method of frequency offset estimation using cross-correlation can precisely estimate Doppler frequency in cell boundary. EKF method estimate the frequency offset and correct the offset using the estimated value at the receiver, and it does not reduce the bandwidth.
efficiency like self-cancellation, as the frequency offset can be estimated from the preamble of the data sequence in each frame.

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